Se puede demostrar que para ondas planas (dependen de una dirección fija en el espacio):

$$\psi(z,t) = \psi_1(z-ct) + \psi_2(z+ct)$$

para ello definimos:

$$\left.\begin{array}{l} u = z - ct \\ v = z + ct \end{array}\right\} \Rightarrow \begin{array}{l} 2z = v+u \Rightarrow z = \dfrac{v+u}{2} \\ 2ct = v-u \Rightarrow ct = \dfrac{v-u}{2} \end{array}$$

Escribo las derivadas para reemplazar en la ec. de ondas:

$$\frac{\partial \psi}{\partial z} = \frac{\partial \psi^{(u,v)}}{\partial u}\frac{\partial u}{\partial z} + \frac{\partial \psi}{\partial v}\frac{\partial v}{\partial z}$$

$$\frac{\partial \psi}{\partial t} = \frac{\partial \psi}{\partial u}\frac{\partial u}{\partial t} + \frac{\partial \psi}{\partial v}\frac{\partial v}{\partial t}$$

pero $\frac{\partial u}{\partial z} = 1 = \frac{\partial v}{\partial z}$ y $\frac{\partial u}{\partial t} = -c$ y $\frac{\partial v}{\partial t} = c$

$$\Rightarrow \frac{\partial \psi}{\partial z} = \frac{\partial \psi}{\partial u} + \frac{\partial \psi}{\partial v} \qquad \frac{\partial \psi}{\partial t} = -c\frac{\partial \psi}{\partial u} + c\frac{\partial \psi}{\partial v}$$

$$\frac{\partial^2 \psi}{\partial z^2} = \frac{\partial^2 \psi}{\partial u^2} + \frac{\partial^2 \psi}{\partial u \partial v} + \frac{\partial^2 \psi}{\partial u \partial v} + \frac{\partial^2 \psi}{\partial v^2} \qquad \left(\frac{\partial}{\partial z} = \frac{\partial}{\partial u} + \frac{\partial}{\partial v}\right) \quad \left(\frac{\partial}{\partial t} = -c\frac{\partial}{\partial u} + c\frac{\partial}{\partial v}\right)$$

$$\frac{\partial^2 \psi}{\partial t^2} = -c\left\{-c\frac{\partial^2 \psi}{\partial u^2} + c\frac{\partial^2 \psi}{\partial u \partial v}\right\} + c\left\{-c\frac{\partial^2 \psi}{\partial u \partial v} + c\frac{\partial^2 \psi}{\partial v^2}\right\}$$

$$= c^2\frac{\partial^2 \psi}{\partial u^2} - c^2\frac{\partial^2 \psi}{\partial u \partial v} - c^2\frac{\partial^2 \psi}{\partial u \partial v} + c^2\frac{\partial^2 \psi}{\partial v^2}$$

pero $c^2\dfrac{\partial^2 \psi}{\partial z^2} = \dfrac{\partial^2 \psi}{\partial t^2} \Rightarrow 2c^2\dfrac{\partial^2 \psi}{\partial u \partial v} = -2c^2\dfrac{\partial^2 \psi}{\partial u \partial v} \Rightarrow$

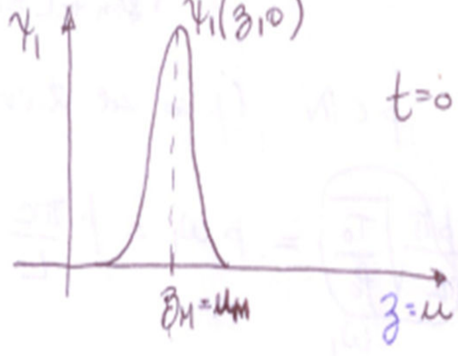
$$\Rightarrow \frac{\partial^2 \psi}{\partial u \partial v} = 0 \Rightarrow \frac{\partial \psi}{\partial u} = \phi_1(u) \overset{\Rightarrow\ \psi_1 = f(u) + c(v)}{} \quad ó \quad \frac{\partial \psi}{\partial v} = \phi_2(v)$$

$$\Rightarrow \psi(u,v) = \psi_1(u) + \psi_2(v)$$

$$\psi(z,t) = \psi_1(z-ct) + \psi_2(z+ct)$$

} estas nuevas sol. son c. l. de ondas estacionarias y viceversa.

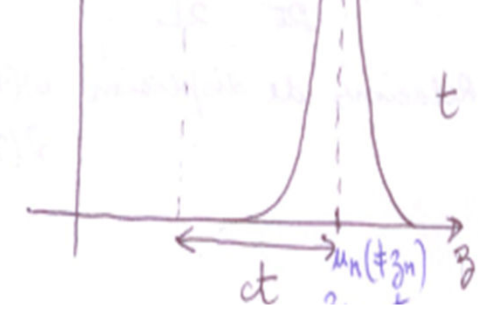
Cada término representa una onda de propagación que viaja con velocidad c.



a $t=0$: $u=z$

$z_M = u_M$ $(t=0)$

u_M: valor de u q' maximiza la función.



$$z_M - ct = u_M \Rightarrow z_M = u_M + ct$$

$\psi_1 \longrightarrow$

$\psi_2 \longleftarrow$